Kinetics of Microbial Growth

Unlimited growth

Assuming t_d = 0.33 h,

in 48 h,

one cell would become 2.33×10^{43} cells

If a cell weighs 10^{-12} g,
then the total would be 2.23×10^{31} g

This would be 4000 times the weight of the earth!

Factors Determining Growth & Synthesis of Products

- **Absolute Factors**

  Nutrients; pH; Temperature; Oxygen

- **Rate-Determining Factors**

  Temperature; pH; Mass Transfer; Energy Transfer

Kinetics of Batch Culture

Growth Rate, r_x

$$= \frac{\Delta x}{\Delta t}$$

or as t becomes infinitesimally small

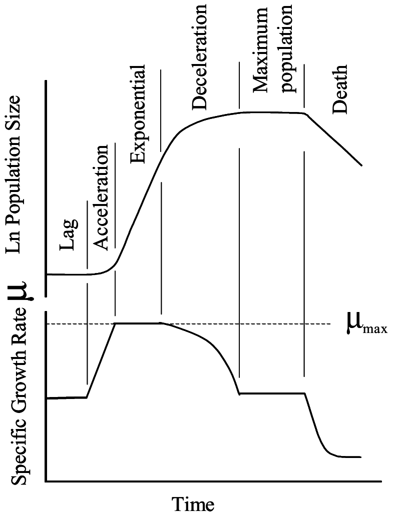
$$= \frac{dx}{dt}$$ Units *e.g.* g cells $\mathrm{mL}^{-1}\mathrm{h}^{-1}$

Specific Growth Rate, μ

$$= \frac{r_x}{x}$$ Units *e.g.* g g^{-1} cells $\mathrm{mL}^{-1}\mathrm{h}^{-1}$

Kinetics of Batch Culture 2

Phases of growth in batch culture



Kinetics of Batch Culture 3

The exponential phase may be described by

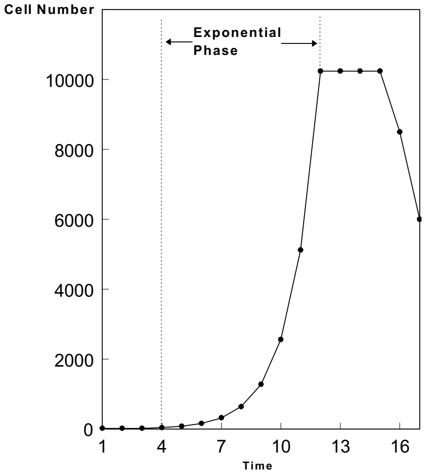
$$\frac{dx}{dt} = \mu x \qquad (2.1)$$

where

x = [microbial biomass]

t = time (h)

μ = specific growth rate (h^{-1})



Kinetics of Batch Culture 4

Integrating equation 2.1 gives

$$x_t = x_0 \, e^{\mu t} \qquad (2.2)$$

where

x_o = [original biomass]

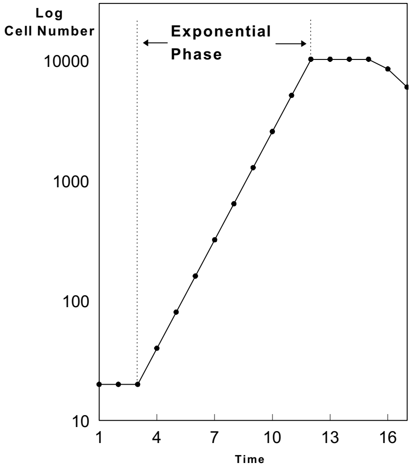
x_t = [biomass after time t]

e = base of natural log

Taking natural log, equation 2.2 becomes

$$\ln x_t = \ln x_0 + \mu t$$

Thus, plot of ln X *vs.* t gives straight line in the exponential phase, slope of which = μ



Determining r_x from data

Time (h)	X (g L^{-1})	S (g L^{-1})
0	0.100	40.00
1	0.134	39.93
2	0.180	39.83
3	0.241	39.70
4	0.323	39.50
5	0.433	39.30
6	0.581	38.97
7	0.778	38.50
8	1.040	38.00
9	1.400	37.20
10	1.870	36.20
11	2.500	34.80
12	3.350	32.90
13	4.490	30.50
14	6.000	27.20
15	8.000	22.80
16	10.70	17.10
17	14.10	9.60
18	17.90	1.11
19	17.90	1.11
20	17.90	1.11

Determining r_x from data 2

Growth Rate, r_x

1. **Hand-drawn tangent**

$$r_x = \frac{y}{x}$$

2. **Numerical differentiation**
   Difference between values on either side of data point

$$r_{x_2} = \frac{x_3 - x_1}{t_3 - t_1}$$

3. **Curve fitting**

$$r_{x_2} = \left[\frac{\ln x_3 - \ln x_1}{t_3 - t_1}\right] \cdot x_2$$

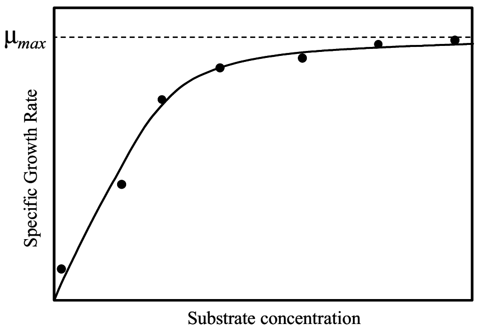
Determining μ from data

Using the values for growth determined as described previously,
Specific Growth Rate may be estimated by the relationship

$$\mu = \frac{r_x}{x}$$

However, in batch culture, μ does not remain constant.

Substrate concentration is a major affecting factor.



Determining μ_{max} from data

1. **By tabulation of values for μ through the exponential phase of the culture.**

Time (h)	x (g L_{-1})	r_x (h^{-1})*	$\mu = r_x/x$	
0	0.100	-	-	
1	0.134	0.040	0.298	
2	0.180	0.054	0.300	← μ_{max}
3	0.241	0.072	0.299	
4	0.323	0.096	0.297	
5	0.433	0.129	0.298	
6	0.581	0.172	0.296	
7	0.778	0.230	0.296	
8	1.040	0.311	0.299	
9	1.400	0.415	0.296	
10	1.870	0.550	0.294	
11	2.500	0.740	0.296	
12	3.350	0.995	0.297	
13	4.490	1.325	0.295	
14	6.000	1.755	0.293	
15	8.000	2.350	0.294	
16	10.70	3.050	0.285	
17	14.10	3.600	0.255	

***r_x via numerical differentiation.**

Determining μ_{max} from data 2

2. **Lineweaver-Burke plot**

$$r_x = \frac{dx}{dt}$$

$$= \frac{\mu_m \cdot S \cdot x}{K_s + S}$$

$$\frac{r_x}{x} = \frac{\mu_m \cdot S}{K_s + S}$$

Taking the reciprocal and since $\frac{rx}{x} = \grave{\imath}$

$$\frac{x}{r_x} = \frac{1}{\mu} = \frac{K_s + S}{\mu_m \cdot S}$$

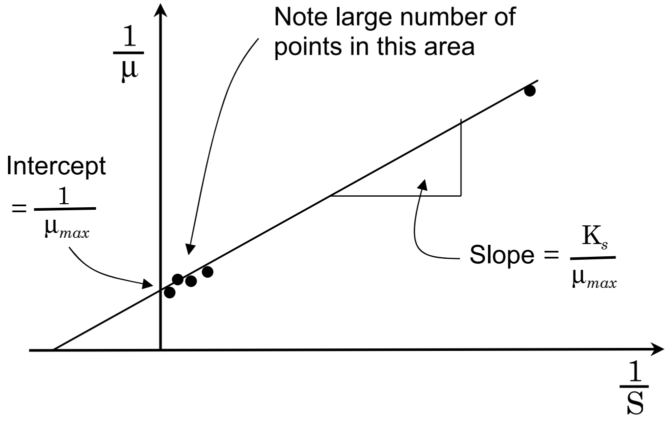
$$= \frac{K_s 1}{\mu_m \cdot S} + \frac{S}{\mu_m \cdot S}$$

$$= \frac{K_s}{\mu_m} \cdot \frac{1}{S} + \frac{1}{\mu_m}$$

Determining μ_{max} from data 3

Compare $\frac{1}{\mu} = \frac{K_S}{\mu_m} \cdot \frac{1}{S} + \frac{1}{\mu_m}$ **with** $y = mx + c$

Thus, when $\frac{1}{\mu}$ **is plotted against** $\frac{1}{S}$**, we get**



Yield factor Y

Y = the ratio of product or cell quantity resulting from a certain quantity of input

e.g. $Y_{x/s}$ Yield of cell weight per unit weight substrate utilized
$Y_{p/n}$ Yield of product weight per unit weight of nitrogen utilized

Determination of Yield Factor on carbon substrate, $Y_{x/s}$

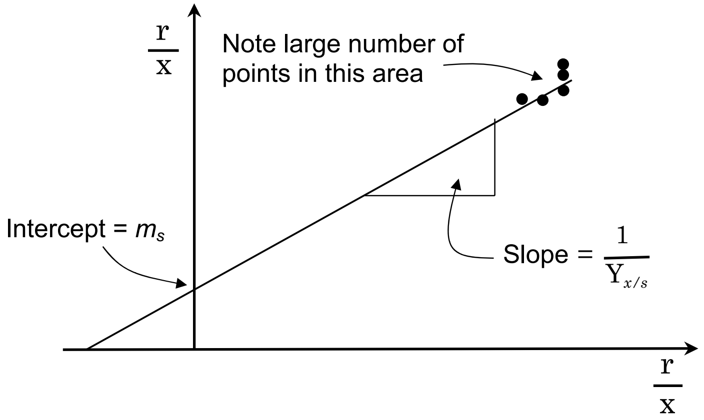
$$r_s = \frac{r_s}{Y_{x/s}} + \frac{m_s}{x}$$

$$\frac{r_s}{x} = \frac{1}{Y_{x/s}} \cdot \frac{r_x}{x} + m_s$$

where r_s = rate of consumption of carbon substrate
m_s = maintenance coefficient on carbon substrate

Determination of Yield Factor on carbon substrate, $Y_{x/s}$ 2

Thus, when $\frac{r_s}{x}$ is plotted against $\frac{r_x}{x}$, we get



The Importance of μ_{max}

For processes where maximal growth rates are diserable, attainment of μ_{max} in culture is important.

Since μ_{max} is determined by the

- genetics of the microorganism
- conditions of culture

Attainment of μ_{max} has implications for both determinants.

For other processes, identification of μ_{max} is important so that it can be avoided
e.g. in the production of secondary metabolites.

The Importance of the Yield Factor Y

Y indicates the degree of efficiency of the conversion of substrates into desired products.

Attainment of efficient Y translates directly into economic efficiency, and thus productivity.

Since Y is determined by the

- genetics of the microorganism
- conditions of culture
- nature of the input (substrate)

Attainment of an efficient Y has implications for the determinants.